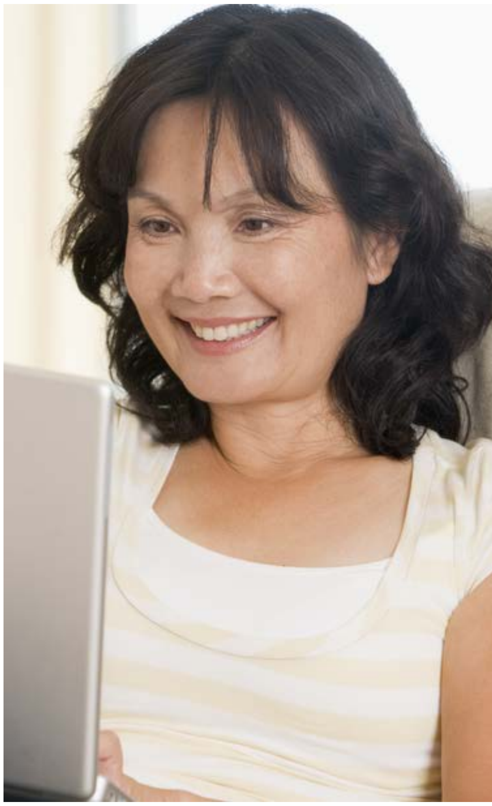




The Building Blocks of Mathematics for Infants and Toddlers: An Annotated Bibliography for Course Developers

2015

Douglas H. Clements, Carrie Germeroth, and Brittany Sovran
Marsico Center for Early Learning and Literacy

Early Educator Central
Pathways to Credentials and Degrees for Infant-Toddler Educators

Contents

Note: This paper was created through Early Educator Central, a web portal federally administered by the U.S. Department of Health and Human Services, Office of Child Care and Office of Head Start, in response to the need for relevant resources to enhance infant-toddler content and coursework. ICF served as the contractor under Contract #HHSP23320095636WC_HHSP2337034T with the Department of Health and Human Services. The views expressed in the document are those of the author and ICF. No official endorsement by the U.S. Department of Health and Human Services is intended or should be inferred.

ICF INTERNATIONAL

ADMINISTRATION FOR CHILDREN & FAMILIES

EarlyEducatorCentral.acf.hhs.gov
Jointly administered and funded by the Office of Child Care and the Office of Head Start

Introduction

Three pictures hang in front of a 6-month-old child. The first shows two dots, the others one dot and three dots, respectively. The infant hears three drumbeats. Her eyes move to the picture with three dots.

At some intuitive level, this infant has recognized number and a change in number. Researchers in many fields have recognized mathematical competencies in children from birth. This is a substantial change from earlier eras dominated by Jean Piaget's (1896-1980) position that children's mathematical thinking occurs only during the later stages of development.

This document presents research on the mathematical competence of infants and toddlers. We organize it in five major sections. We begin by describing our methodology. Then the research is presented in five domains: spatial relationships, number sense, problem solving and prediction, the role of caregivers (parents and providers), and integrating math across the day.

Methodology

We conceptualized mathematics for infants and toddlers by looking at the commonly held standards for this age group. In most state standards, the foundations or building blocks of mathematics are found within a "Cognitive Development" domain. Most states include the topics of Spatial Relationships and Number Sense within this domain.[1] Indicators within these topics generally align with the Mathematics and Problem Solving domains that are more commonly used in early childhood (birth to five years) and throughout K–12. Thus, our annotated bibliography is organized primarily around the topics of Spatial Relationships and Number Sense. We also include articles pertaining to Patterning because it is an important general mathematical process[2], the subject of multiple studies, and a topic in many curricula. Finally, after reviewing all articles, additional categories were added pertaining to the impact of caregivers (parents, providers/ teachers) on young children's math development, and integrating math throughout the day.

Because we were interested in these domains primarily, we conducted our literature review by entering the keywords *spatial*, *number, infant, toddler*, and *patterning*. Using these search terms, we searched the Academic Search Complete and ERIC (Education Resources Information Center) databases. This search strategy, which aimed to find both studies conducted in the United States and internationally, was limited to the English language. The electronic searches were supplemented by checking the reference lists of included articles, existing systematic reviews and meta-analyses, and hand-searching online databases of research. We focused additional searches on practitioner-oriented journals such as *Teaching Young Children, Early Education and*

---

[1] Scott-Little, C., Kagan, S. L., Reid, J. L., & Castillo, E. (2012). Early mathematics standards in the United States: The quest for alignment. Los Altos, CA: Heising-Simons Foundation.

[2] National Research Council. (2009). Mathematics in early childhood: Learning paths toward excellence and equity. Washington, DC: National Academy Press.

Development, Young Children, Childhood Education, Teaching Children Mathematics, and Early Childhood Today. Finally, researchers also reviewed articles cited in recently developed and research-based state standards documents such as the *Colorado Early Learning and Development Guidelines and the California Infant/Toddler Learning & Development Foundations*.

Taken together, these search strategies yielded 1,194 references. Once we removed the duplicates, we conducted an initial screening of titles and abstracts, and marked articles for inclusion or exclusion. Those relevant to the purpose of this annotated bibliography were included for full text review. Conflicting inclusion or exclusion decisions were resolved through discussion and, as a result of this process, the team decided that 53 articles met all criteria and were included. The most common reasons for the title, abstract, or full text to be considered irrelevant were related to the age of the study participants (e.g., older students as opposed to children ages 0–3 years) and to the topic of interest and outcome measures (e.g., only language or socio-emotional development as opposed to math).

Quick Reference Table

Topic Area	Age Group	Document Type	Full Citation
Spatial Relationships	7 months old	Empirical Research	Baillargeon, R. (1987). Young infants' reasoning about the physical and spatial properties of a hidden object. *Cognitive Development, 2*(3), 179–200. doi:10.1016/S0885-2014(87)90043-8
Spatial Relationships (also see Number Sense)	Early Childhood	Book	Clements, D. H., Sarama, J., & DiBiase, A.-M. (2004). Engaging young children in mathematics: Standards for early childhood mathematics education. Mahwah, NJ: Erlbaum.
Spatial Relationships (also see Number Sense)	2–8 years old	Practical Tips/ Research	Clements, D. H., & Sarama, J. (2014). *Learning and teaching early math: The learning trajectories approach* (2nd ed.). New York City: Routledge.
Spatial Relationships (also see Number Sense, Problem Solving and Prediction, Caregivers)	Early Childhood	Book	Sarama, J., & Clements, D. H. (2009). Early childhood mathematics education research: Learning trajectories for young children. New York City: Routledge.
Spatial Relationships	6.5 months old	Empirical Research	Duffy, S., Huttenlocher, J., Levine, S., & Duffy, R. (2005). How infants encode spatial extent. Infancy, 8(1), 81–90. doi:10.1207/s15327078in0801_5
Spatial Relationships, Number Sense	Infants/Toddlers	Practical Tips/ Research	Greenberg, J. (2012). More, all gone, empty, full: Math talk every day in every way. *Young Children, 67*(3), 62–64. Retrieved from https://www.naeyc.org/yc/files/yc/file/201205/RockingAndRolling_YC0512.pdf
Spatial Relationships	Infants/Toddlers	Book	Newcombe, N. S., & Huttenlocher, J. (2000). *Making space: The development of spatial representation and reasoning.* Cambridge, MA: MIT Press.

Topic Area	Age Group	Document Type	Full Citation
Spatial Relationships	14–24 months old	Empirical Research	Örnkloo, H., & von Hofsten, C. (2007). Fitting objects into holes: On the development of spatial cognition skills. *Developmental Psychology, 43*(2), 404–416. doi:10.1037/0012-1649.43.2.404
Number Sense	Newborns	Empirical Research	Antell, S. E., & Keating, D. P. (1983). Perception of numerical invariance in neonates. *Child Development, 54*(3), 695–701.
Number Sense	Toddlers	Empirical Research	Baroody, A. J., Li, X., & Lai, M.-I. (2008). Toddlers' spontaneous attention to number. *Mathematical Thinking and Learning, 10*(3), 240–270. doi:10.1080/10986060802216151
Number Sense	3–5 years old	Empirical Research	Benoit, L., Lehalle, H., & Jouen, F. (2004). Do young children acquire number words through subitizing or counting? *Cognitive Development, 19*(3), 291–307. doi:10.1016/j.cogdev.2004.03.005
Number Sense	9–11 months old	Empirical Research	Brannon, E. M. (2002). The development of ordinal numerical knowledge in infancy. *Cognition, 83*(3), 223–240. doi:10.1016/S0010-0277(02)00005-7
Number Sense	Infants	Empirical Research	Cantrell, L., & Smith, L. B. (2013). Open questions and a proposal: A critical review of the evidence on infant numerical abilities. *Cognition, 128*(3), 331–352. doi:10.1016/j.cognition.2013.04.008
Number Sense	8, 24, 48 months old	Empirical Research	Ceulemans, A., Titeca, D., Loeys, T., Hoppenbrouwers, K., Rousseau, S., & Desoete, A. (2015). The sense of small number discrimination: The predictive value in infancy and toddlerhood for numerical competencies in kindergarten. *Learning and Individual Differences, 39*, 150–157. doi:10.1016/j.lindif.2015.03.009
Number Sense	6–8 months old	Empirical Research	Clearfield, M. W., & Mix, K. S. (1999). Number versus contour length in infants' discrimination of small visual sets. *Psychological Science, 10*(5), 408–411.
Number Sense	Early Childhood	Review	Clements, D. H. (1989). Consensus, more or less. *Journal for Research in Mathematics Education, 20*(1), 111–119.
Number Sense	10–12 months old	Empirical Research	Feigenson, L., Carey, S., & Hauser, M. (2002). The representations underlying infants' choice of more: Object files versus analog

Topic Area	Age Group	Document Type	Full Citation
			magnitudes. *Psychological Science, 13*(2), 150–156. doi:10.1111/1467-9280.00427
Number Sense	Infants	Empirical Research	Feigenson, L., Carey, S., & Spelke, E. (2002). Infants' discrimination of number versus continuous extent. *Cognitive Psychology, 44*(1), 33–66. doi:10.1006/cogp.2001.0760
Number Sense	2–8 years old	Book	Fuson, K. C. (1988). *Children's counting and concepts of number*. New York City: Springer-Verlag.
Number Sense	Early Childhood	Empirical Research	Gallistel, C. R., & Gelman, R. (1992). Preverbal and verbal counting and computation. *Cognition, 44*(1–2), 43–74. doi:10.1016/0010-0277(92)90050-R
Number Sense (also see Integrating Math Across the Day)	Early Childhood	Practical Tips/ Research	Geist, K., & Geist, E. (2008). Do re mi, 1-2-3: That's how easy math can be—Using music to support emergent mathematics. *Young Children, 63*(2), 20–25. Retrieved from http://0-search.proquest.com.bianca.penlib.du.edu/docview/61969123?accountid=14608[3]
Number Sense	Early Childhood	Book	Gelman, R., & Gallistel, C. R. (1978). *The child's understanding of number*. Cambridge, MA: Harvard University Press.
Number Sense	Infants, Toddlers	Practical Tips/Research	Greenberg, J. (2012). More, all gone, empty, full: Math talk every day in every way. Young Children, 67(3), 62–64. Retrieved from https://www.naeyc.org/yc/files/yc/file/201205/RockingAndRolling_YC0512.pdf
Number Sense	5–6 months old	Empirical Research	Jordan, K. E., Suanda, S. H., & Brannon, E. M. (2008). Intersensory redundancy accelerates preverbal numerical competence. *Cognition, 108*(1), 210–221. doi:10.1016/j.cognition.2007.12.001
Number Sense	2.5–3.5 years old	Empirical Research	Li, X., & Baroody, A. J. (2014). Young children's spontaneous attention to exact quantity and verbal quantification skills.

[3] Please note that free access is not available to this document through the cited website.

Topic Area	Age Group	Document Type	Full Citation
			European Journal of Developmental Psychology, 11(5), 608–623. doi:10.1080/17405629.2014.896788
Number Sense (also see Caregivers)	32–38 months old	Empirical Research	Linnell, M., & Fluck, M. (2001). The effect of maternal support for counting and cardinal understanding in pre-school children. *Social Development, 10*(2), 202–220. doi:10.1111/1467-9507.00159
Number Sense	6 months old	Empirical Research	Lipton, J. S., & Spelke, E. S. (2003). Origins of number sense: Large-number discrimination in human infants. *Psychological Science*, *14*(5), 396–401.
Number Sense	Early Childhood	Empirical Research	Mazzocco, M. M. M., Feigenson, L., & Halberda, J. (2011). Preschoolers' precision of the approximate number system predicts later school mathematics performance. *PLoS ONE, 6*(9), 1–8. doi:10.1371/journal.pone.0023749
Number Sense	Birth to School Entry	Empirical Research	Mix, K. S., Huttenlocher, J., & Levine, S.C. (2002). *Quantitative development in infancy and early childhood.* New York City: Oxford University Press.
Number Sense	2–4 years old	Empirical Research	Sarnecka, B. W., & Lee, M. D. (2009). Levels of number knowledge during early childhood. *Journal of Experimental Child Psychology, 103*(3), 325–337. doi:10.1016/j.jecp.2009.02.007
Number Sense	6–8 months old	Empirical Research	Starkey, P., & Cooper, R. G. (1980). Perception of numbers by human infants. *Science, 210*(4473), 1033–1035. doi:10.1126/science.7434014
Number Sense	6–8 months old	Empirical Research	Starkey, P., Spelke, E. S., & Gelman, R. (1990). Numerical abstraction by human infants. *Cognition, 36*(2), 97–127. doi:10.1016/0010-0277(90)90001-Z
Number Sense	10–12 months old	Empirical Research	Strauss, M. S., & Curtis, L. E. (1981). Infant perception of numerosity. *Child Development, 52*(4), 1146–1152.
Number Sense	4–5 months old	Empirical Research	Wynn, K. (1992). Addition and subtraction by human infants. *Nature, 358*(6389), 749–750. doi:10.1038/358749a0

Topic Area	Age Group	Document Type	Full Citation
Number Sense	2–3 years old	Empirical Research	Wynn, K. (1992). Children's acquisition of the number words and the counting system. *Cognitive Psychology, 24*(2), 220–251. doi:10.1016/0010-0285(92)90008-P
Number Sense	Infants	Empirical Research	Wynn, K. (1998). Numerical competence in infants. In C. Donlan (Ed.), *The development of mathematical skills* (pp. 3–25). Hove, England: Psychology Press/Taylor & Francis.
Number Sense	Infants	Empirical Research	Wynn, K. (1998). Psychological foundations of number: Numerical competence in human infants. *Trends in Cognitive Sciences, 2*(8), 296–303. doi:10.1016/S1364-6613(98)01203-0
Number Sense	7 months old	Empirical Research	Xu, F. (2003). Numerosity discrimination in infants: Evidence for two systems of representations. *Cognition, 89*(1), B15–B25. doi:10.1016/S0010-0277(03)00050-7
Number Sense	8 months old	Empirical Research	Xu, F., & Spelke, E. S. (2000). Large number discrimination in 6-month-old infants. *Cognition, 74*(1), B1–B11. doi:10.1016/S0010-0277(99)00066-9
Number Sense	9 months old	Empirical Research	Xu, F., Spelke, E. S., & Goddard, S. (2005). Number sense in human infants. *Developmental Science, 8*(1), 88–101. doi:10.1111/j.1467-7687.2005.00395.x
Problem Solving & Prediction	Infants	Empirical Research	Denison, S. M. (2012). *Inductive Inference in Infants and Young Children: The Role of Probabilistic Reasoning*. (3555645 Ph.D.), University of California, Berkeley, Ann Arbor. Retrieved from http://escholarship.org/uc/item/4ff9g022#page-1
Problem Solving & Prediction	12–14 months old	Empirical Research	Denison, S., & Xu, F. (2010). Twelve- to 14-month-old infants can predict single-event probability with large set sizes. *Developmental Science, 13*(5), 798–803. doi:10.1111/j.1467-7687.2009.00943.x
Problem Solving & Prediction	3 years old	Empirical Research	Richards, D. D., & Siegler, R. S. (1981). Very young children's acquisition of systematic problem-solving strategies. *Child Development, 52*(4), 1318–1321.

Topic Area	Age Group	Document Type	Full Citation
Problem Solving & Prediction	12 months old	Empirical Research	Téglás, E., Girotto, V., Gonzalez, M., & Bonatti, L. L. (2007). Intuitions of probabilities shape expectations about the future at 12 months and beyond. *Proceedings of the National Academy of Sciences of the United States of America, 104*(48), 19156–19159. doi:10.1073/pnas.0700271104
Problem Solving & Prediction	8 months old	Empirical Research	Xu, F., & Denison, S. (2009). Statistical inference and sensitivity to sampling in 11-month-old infants. *Cognition, 112*(1), 97–104. doi:10.1016/j.cognition.2009.04.006
Caregivers (Parents and Providers)	Toddlers	Book	Cook, G. A., Roggman, L. A., & Boyce, L. K. (2011). Fathers' and mothers' cognitive stimulation in early play with toddlers: Predictors of 5th grade reading and math. *Family Science, 2*(2), 131–145. doi:10.1080/19424620.2011.640559
Caregivers (Parents and Providers)	14–46 months old	Empirical Research	Gunderson, E. A., & Levine, S. C. (2011). Some types of parent number talk count more than others: Relations between parents' input and children's cardinal-number knowledge. *Developmental Science, 14*(5), 1021–1032. doi:10.1111/j.1467-7687.2011.01050.x
Caregivers (Parents and Providers)	14–46 months old	Empirical Research	Levine, S. C., Suriyakham, L. W., Rowe, M. L., Huttenlocher, J., & Gunderson, E. A. (2010). What counts in the development of young children's number knowledge? *Developmental Psychology, 46*(5), 1309–1319. doi:10.1037/a0019671
Caregivers (Parents and Providers) (also see Number Sense)	32–38 months old	Empirical Research	Linnell, M., & Fluck, M. (2001). The effect of maternal support for counting and cardinal understanding in pre-school children. *Social Development, 10*(2), 202–220. doi:10.1111/1467-9507.00159
Caregivers (Parents and Providers)	2–3 years old	Empirical Research	McKelvey, L. M., Bokony, P. A., Swindle, T. M., Conners-Burrow, N. A., Schiffman, R. F., & Fitzgerald, H. E. (2011). Father teaching interactions with toddlers at risk: Associations with later child academic outcomes. *Family Science, 2*(2), 146–155. doi:10.1080/19424620.2011.637710

Topic Area	Age Group	Document Type	Full Citation
Integrating Math Across the Day (also see Number Sense)	Early Childhood	Practical Tips/ Research	Geist, K., & Geist, E. (2008). Do re mi, 1-2-3: That's how easy math can be—Using music to support emergent mathematics. *Young Children, 63*(2), 20–25. Retrieved from http://0-search.proquest.com.bianca.penlib.du.edu/docview/61969123?accountid=14608[4]
Integrating Math Across the Day (also see Number Sense)	Early Childhood	Practical Tips/ Research	Geist, K., Geist, E. A., & Kuznik, K. (2012). The patterns of music: Young children learning mathematics through beat, rhythm, and melody. *Young Children, 67*(1), 74–79. Retrieved from https://www.naeyc.org/files/yc/file/201201/Geist_Patterns_of_Music_Jan012.pdf
Integrating Math Across the Day (also see Problem Solving & Prediction)	Early Childhood	Practical Tips/ Research	Honig, A. (2007). Play: Ten power boosts for children's early learning. *Young Children, 62*(5), 72–78. Retrieved from https://mymission.lamission.edu/userdata/mermelrd/docs/NAE51.pdf
Integrating Math Across the Day	Early Childhood	Empirical Research	Powell, S. R., & Nurnberger-Haag, J. (2015). Everybody counts, but usually just to 10! A systematic analysis of number representations in children's books. *Early Education and Development, 26*(3), 377–398. doi:10.1080/10409289.2015.994466
Integrating Math Across the Day	Early Childhood	Practical Tips/ Research	Reifel, S. (1984). Block construction: Children's developmental landmarks in representation of space. *Young Children, 40*(1), 61–67.
Integrating Math Across the Day	1–5 years old	Empirical Research	Sumpter, L., & Hedefalk, M. (2015). Preschool children's collective mathematical reasoning during free outdoor play. *The Journal of Mathematical Behavior, 39*, 1–10. doi:10.1016/j.jmathb.2015.03.006

[4] Please note that free access is not available to this document through the cited website.

Spatial Relationships

Introduction[5]

Spatial orientation involves understanding and operating on relationships between different positions in space, at first with respect to one's own position and movement through it, and eventually from a more abstract perspective that includes maps. Like number, spatial orientation has been postulated as a core domain—that is, a domain of development for which competencies are present from birth. For example, infants focus their eyes on objects and then begin to follow moving objects. Toddlers ignore other cues and instead use geometric information about the overall shape of their environment to solve location tasks. Early developing or inborn endowments interact with experience and sociocultural influences to affect development.

As the resources presented below show, young children can reason about spatial perspectives and spatial distances, although their abilities develop considerably throughout the early childhood years. As an example, in the first year of life, infants can perceive the shape and size of objects and can represent the location of objects in a three-dimensional space.

Children develop different ways to navigate space. In the first year of life, stationary infants rely on "response learning" to locate objects in their environment. An example is children who incorrectly turn in the same direction that originally led to a toy even after being physically reoriented. During their second year, children develop the ability to code locations using objects in their external environment. They also become capable of *spatial reasoning*, in that they can solve problems with that information. Another later developing way in which to navigate space is called "place learning." In contrast to Piaget's notion that children develop a "Euclidean system" only by age 9 or 10, research indicates that toddlers are able to code distance information and use that to locate objects.

Educational experiences can help children of almost any age develop spatial skills. Spatial training in identifying a fixed location from two orientations is effective with infants as young as 6 (but not 4) months of age.

Although infants are endowed with potential spatial competencies, experience-expectant processes engender their development. That is, experiences with space are sufficiently similar across cultures and individuals that developmental processes are almost guaranteed to have certain environmental inputs. Thus, inborn potentialities are no more or less fundamental than these expected environmental inputs; rather, the key is in their interaction. In summary, the infant

---

[5] All introductions are based on two books that review the research on early mathematics and its practical application:

Clements, D. H., & Sarama, J. (2014). *Learning and teaching early math: The learning trajectories approach* (2nd ed.). New York City: Routledge.

Sarama, J., & Clements, D. H. (2009). Early childhood mathematics education research: Learning trajectories for young children. New York City: Routledge.

possesses biologically provided tools, but these inborn competencies or foundations guide, but do not determine, development; instead, they interact with maturation and physical and social experience, with the role of these differing, depending on the subdomain.

Research indicates that even infants can have perception of shapes, such as wholes, as well as parts, of geometric patterns. Children as young as 3 years of age engage in active spatial analysis in both construction and perception, which is a process that changes with development.

Infants may be born with a tendency to form certain mental prototypes of shape. When people in a primitive culture with no explicit geometric concepts were asked to choose a "best example" of a group of shapes, such as a group of quadrilaterals and near-quadrilaterals, they chose a square and a circle more often, even when close variants ("not quite a circle") were in the group. For example, the group with squares included square-like shapes that were not closed, had curved sides, and had non-right angles. So, all people might have innate preferences for closed, symmetrical shapes. And infants as young as 4 months dishabituate more quickly to symmetrical figures than asymmetrical figures, at least for vertical symmetry. A preference for vertical symmetry seems to develop between 4 and 12 months of age; vertical bilateral symmetry remains preferred by children over horizontal symmetry, which, in turn, is more preferred than diagonal symmetries.

Geometric thinking is an essential human ability that contributes to mathematical ability. Its importance is highlighted by findings that infants assign greater weight to spatiotemporal information than color or form in their definition of what an object is. Thereafter, toddlers use the shape of objects as the essential cue to learn the identity and names of objects. For example, supporting 17-month-olds to attend to shape led them to generalize that objects with similar shapes have the same name and engendered a dramatic increase of 350 percent in learning new words outside of the research laboratory.

Block building provides a view of children's initial abilities to perform shape composition with 3-D objects. Children initially build structures from simple components and later explicitly synthesize 3-D shapes into higher-order 3-D shapes. In their first year, children either engage in little systematic organization of objects or show little interest in stacking. Instead, they pound, clap together, slide the blocks, or use single blocks to represent an object, such as a house or vehicle. Children's first composite constructions are simple combinations of pairs. Even when combining more than two blocks, children remain in one dimension. Stacking begins at 1 year of age, thus showing the use of the spatial relationship of "on." Occasionally, children balance blocks intuitively as a stack of rectangular prisms, for example, but often they place a block off center or on an edge of a triangular prism. In the latter case, they recognize their lack of success when the block falls, but do not attempt to understand it. The "next to" relation, creating structures such as a road or "train" of blocks, develops at about 1.5 years of age.

At about 2 years of age, children place each successive block congruently on or next to the one previously placed. They appear to recognize that blocks stacked vertically do not fall when so placed. At this point, children begin to reflect and anticipate. Around 2 to 3 years of age, children begin to extend their building to two dimensions, covering, to an extent, a plane in creating a floor or wall.

Children's understanding of measurement is rooted in infancy and the preschool years, but grows over many years, as the work of Piaget and his collaborators has shown. As with number, however, Piagetians underestimated the abilities of the youngest children. For example, shown an object such as a ball that was then hidden by a drawbridge, infants looked longer when the drawbridge rotated past the point where the ball should have stopped the drawbridge from rotating. This demonstrates that infants are sensitive to continuous quantity (i.e. that the ball should have stopped the drawbridge) and comparisons and accumulations of continuous quantity, at least under some conditions. These studies show that the early cognitive foundations of mathematics are not limited or unique to number. As with number, however, these abilities have limits. Infants and toddlers can discriminate between lengths of dowels, but only when a salient standard was present; 4-year-olds could discriminate with or without such as standard.

References on Spatial Relations

Baillargeon, R. (1987). Young infants' reasoning about the physical and spatial properties of a hidden object. ***Cognitive Development*****, 2(3), 179–200. doi:10.1016/S0885-2014(87)90043-8**

This article presents two experiments conducted with 7-month-old infants to determine their ability to represent and reason about the physical and spatial properties of a hidden (i.e. occluded) object. The first experiment was designed to test whether 7-month-old infants are able to represent and reason about location and the dimensions of an occluded object. In this experiment, two groups of infants were tested. One group saw a screen that rotated upward and then slid backward. The results indicated that infants expected the screen to stop sliding sooner when an object was 10 cm, rather than 25 cm, behind it. This suggests that the infants were able to represent the location of the object behind the screen and use this information to estimate at what point the screen should reach the object and stop. The second experiment was designed to test whether 7-month-old infants were able to represent and to reason about the compressibility of an occluded object. In this experiment, the infants saw a screen that rotated upward and then backward toward either a soft or hard object. The results indicated that the infants expected the screen to stop sooner when an incompressible (i.e. hard) object stood behind the screen rather than a compressible (i.e. soft) object. This finding suggests that infants represented the height and the compressibility of the object behind the screen and used this information to determine at what point the screen should reach the object and whether it could continue rotating at this point. The results of these two experiments indicate that 7-month-old infants can represent and reason about the physical and spatial properties of an occluded object.

Clements, D. H., Sarama, J., & DiBiase, A.-M. (2004). Engaging young children in mathematics: Standards for early childhood mathematics education. Mahwah, NJ: Erlbaum.

This book, a report of a conference on early math standards funded by the National Science Foundation and the Exxon-Mobil Foundation, constitutes a set of guidelines that enable standards writing groups to create consistent and complementary (and, as much as possible, common) standards that are based on a current understanding of research, practice, and policy in early childhood mathematics education. The first section, “Major Themes and Recommendations,” summarizes the research findings and guidelines. Some of the conclusions relevant to the youngest children include the following (we include number especially because research shows that it has a large spatial component in very young children’s cognition):

- Infants and toddlers spend a great deal of time exploring space and learning about the properties and relations of objects in space.
- The chapter “Geometric and Spatial Thinking in Early Childhood Education” elaborates on research findings in the domains of spatial reasoning, knowledge of shapes, and measurement. Some of the findings relevant to the youngest children include the following:
- In the first year of life, infants can perceive the shape and size of objects and can represent the location of objects in a 3-D space.
- Toddlers learn to use spatial vocabulary such as *over*, *under*, *above*, *on*, *beside*, *next to*, and *between*.
- Toddlers and 3-year-olds can place objects in specified locations near distant landmarks, but “lose” locations that are not specified ahead of time once they move. They may be able to form simple frameworks, such as the shape of the arrangement of several objects, which has to include their own location.
- Toddlers can create simple pictures from shapes. They may informally create 2-D shapes and 3-D buildings that have linear or rotational symmetry.
- Toddlers can match shapes first with the same size and orientation, then with different sizes and orientations.

Clements, D. H., & Sarama, J. (2014). *Learning and teaching early math: The learning trajectories approach***. (2nd ed.). New York City: Routledge.**

Sarama, J., & Clements, D. H. (2009). Early childhood mathematics education research: Learning trajectories for young children. New York City: Routledge.

These books review research on mathematics, thinking, learning, and development for children from birth to age 8. The relevant chapters for spatial relations include chapters 7–12 in both books (they are complementary chapters—the 2009 book emphasizes detailed descriptions of the research and the 2014 book summaries this and emphasizes teaching and curricular activities), as well as pages 257–260, 263, 274, and 300–301 in the 2014 edition. Included within these books are charts with developmentally appropriate activities for children across each learning trajectory. Thus, readers can see the developmental level of thinking *and* the associated activities for each age. Each chapter (especially in the 2014 book) provides practical tips and strategies for teachers. The highlights of these chapters are in the introduction to this section.

Duffy, S., Huttenlocher, J., Levine, S., & Duffy, R. (2005). How infants encode spatial extent. *Infancy, 8***(1), 81–90. doi:10.1207/s15327078in0801_5**

In this study, the researchers sought to better understand how 6.5-month-old infants encode an object's spatial extent. This study involved habituating 6.5-month-old infants to a dowel inside of a container, and then altering the size of the container and dowel in order to determine whether the infants dishabituate when the relation between the dowel and the container was held constant. The researchers also tested whether infants dishabituate to a relation change between the dowel and the container by altering the size of the container but not the dowel. The results indicated that infants only dishabituated when the relation changed. This suggests that infants encode the relation between two objects rather than the absolute size of a single object.

Greenberg, J. (2012). More, all gone, empty, full: Math talk every day in every way. *Young Children, 67***(3), 62–64. Retrieved from**
https://www.naeyc.org/yc/files/yc/file/201205/RockingAndRolling_YC0512.pdf

This brief and practical article provides suggestions for how teachers and caregivers can incorporate math talk with infants and toddlers in everyday living. The author gives examples of how to pull in the math concepts of number and operation; shapes and spatial relationships; measurement; patterns, relationships, and change; and collecting and organizing information into everyday activities and conversations. Specific to spatial relationships, this article discusses ways that teachers and/or caregivers can foster shape and geometry development by recognizing, naming, comparing, and contrasting objects based on their geometric appearance, as well as understanding the physical relationship between self and objects.

Newcombe, N. S., & Huttenlocher, J. (2000). Making space: The development of spatial representation and reasoning. Cambridge, MA: MIT Press.

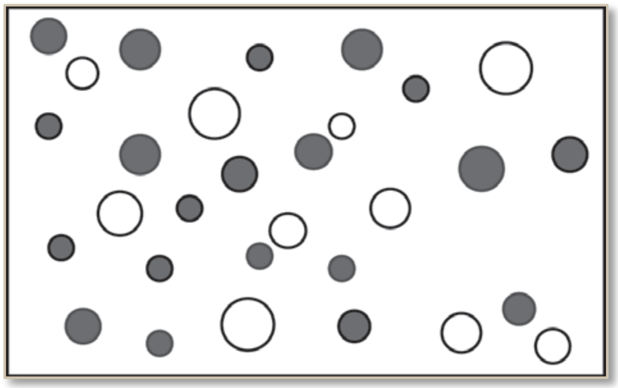
This book focuses on the human development of spatial representation and reasoning. The authors explore separate domains of spatial development with the aim of understanding how spatial development informs developmental theory. Chapters in this book delve into how we think about space, hypotheses about infant location coding, automaticity, important questions about the development of location coding, the development of spatial thought, models and maps, space and language, and thinking about development.

Örnkloo, H., & von Hofsten, C. (2007). Fitting objects into holes: On the development of spatial cognition skills. *Developmental Psychology*, *43***(2), 404–416. doi:10.1037/0012-1649.43.2.404**

This research examined 14- to 26-month-old infants' understanding of the spatial relationships between objects and apertures in an object manipulation task (i.e., a shape-sorting box toy). The task was to insert objects with various cross-sections (circular, square, rectangular, ellipsoid, and triangular) into fitting apertures. The infant was required to mentally rotate the object to be fit into the aperture and use that information to plan the action. The object was presented standing up in half of the trials; in the other half, it was lying down. The results showed that infants solved the problem consistently from age 22 months and that a successful solution was associated with appropriate preadjustments before the hand arrived with the block to the aperture. No sex differences were found.

Number Sense

Introduction



As the vignette that begins this document suggests, children possess a variety of mathematical competencies at or soon after birth. Most children without specific disabilities have impressive sensitivity to number. For example, 6-month-olds can discriminate the 1:2 ratio (as in the figure to the right), but by 9 months of age, they can also distinguish sets in a 2:3 ratio (e.g., 10 compared to 15).

These abilities appear to form one of the innate, foundational abilities for all later numerical knowledge. The better children are with such tasks, the better they do with mathematics in preschool—much more strongly than for those who are low in mathematical knowledge. Thus, children begin to compare quantities innately very early and establish simple correspondences intuitively as early as 20 months of age, and at most by 24 months of age. Of course, these early competencies undergo considerable development throughout the early childhood years.

By 24 months of age, many toddlers have learned one or more number words and they begin to try to count using verbal number names as early as age 2 or 3. Again, surprisingly, infants and toddlers show initial competence in arithmetic. For example, they notice the effects of increasing or decreasing small collections by one item. It is important to note that such behaviors do not imply that these very young children have conceptions of number like older students or adults. However, they certainly possess foundational abilities upon which parents and providers can build.

References on Number Sense

Antell, S. E., & Keating, D. P. (1983). Perception of numerical invariance in neonates. *Child Development, 54***(3), 695–701.**

This study aimed to test the hypothesis that cognitive development is a continuous process, and that newborns can demonstrate precursors of later abilities when performance factors are controlled. This study evaluated 40 newborn infants ranging in age from 21 hours to 144 hours old on their ability to discriminate among visual stimulus arrays consisting of two versus three or four versus six black dots. Using the habituation/recovery technique of observing infant attention as an index of complex processing, infants were able to detect numerical difference for the small number sets (two to three and three to two) but not for the larger sets (four to six and six to four). These results suggest that the ability of newborns to differentiate between small numerical arrays in a habituation/dishabituation paradigm may be evidence of complex information processing during the first week of life.

Baroody, A. J., Li, X., & Lai, M.-l. (2008). Toddlers' spontaneous attention to number. *Mathematical Thinking and Learning, 10***(3), 240–270. doi:10.1080/10986060802216151**

The aim of this study was to examine whether and how age, the size of a collection, and the makeup of a collection affect a toddler's tendency for spontaneous attention to number. Theoretical rationale and existing evidence for nonverbal and verbal number development in infants and toddlers are presented. The study involved 37 toddlers between the ages of 2 and 4.25 years being shown a nonverbal matching and nonverbal production version of spontaneous attention to number tasks. Each version involved the same 36 trials, which were comprised of 12 different collections of two, three, and four items each. The items were either (a) homogenous, (b) the same shape but two different colors, or (c) two different shapes and two different colors. The results indicated that toddlers are able to more accurately reproduce numbers on the matching version than on versions that required forming a mental image. Additionally, age, size, and makeup had a significant effect on participants' tendency to attend to number. The significant drop-off in this tendency with collections of more than two items is inconsistent with the nativists' (who argue that humans are born with nonverbal counting principles and concepts of numbers) hypothesis of an innate cardinal concept of three.

Benoit, L., Lehalle, H., & Jouen, F. (2004). Do young children acquire number words through subitizing or counting? *Cognitive Development, 19***(3), 291–307. doi:10.1016/j.cogdev.2004. 03.005**

The aim of this study was to explore how young children acquire the cardinal meaning of small-number words. The researchers present two differing hypotheses: the first suggests that children perform better when items are presented in succession and the second suggests that subitizing predicts better performance when items are presented simultaneously. This study involved 48 children ranging in age from 3 to 5 years being shown collections of red dots ranging from one dot to six dots. The children were divided into either a simultaneous task group or a consecutive task group. In the simultaneous task, children were shown red dots on a computer screen and asked to verbally report how many dots appeared. In the consecutive task group, children were shown red dots on a computer screen one at a time, asked to count them, and then verbally report how many dots appeared. Additionally, children were shown both familiar and unfamiliar configurations of the dots. Results indicated that, for small numbers, 3-year-old children perform better on short, simultaneous presentations of dots than one-by-one presentations of the same items. Additionally, children performed better when the dots were presented in familiar configurations, except for small numbers in simultaneous presentation. In general, these results support the hypothesis that subitizing is the developmental pathway for acquiring the meaning of the first few number words because it allows the child to grasp the whole and the elements at the same time.

Brannon, E. M. (2002). The development of ordinal numerical knowledge in infancy. ***Cognition, 83*****(3), 223–240. doi:10.1016/S0010-0277(02)00005-7**

This study discusses various experiments that explore the differences between 11-month-old and 9-month-old infants in discriminating numerical sequences that descend in numerical value versus increase in numerical value. In the first experiment, infants were randomly assigned to be habituated to either ascending or descending numerical sequences. Next, they were tested with both ascending and descending sequences. The results indicated that the 11-month-old infants were able to represent the change in ordinal relations (i.e., ascending to descending and vice versa) between numerical values and that the 9-month-old infants were not able to do so. Experiment 2 was conducted similarly to Experiment 1; however, element size, cumulative surface area, and the density of the trials were controlled. The results from Experiment 2 echoed the results found in Experiment 1: 11-month-old infants were able to detect a change in ordinal direction whereas 9-month-old infants were not able to do so. In Experiment 3, the 9-month-old infants were tested in another version of the same task where displays differed in the size of a single square on the computer monitor rather than in number. The results of Experiment 3 indicated that the 9-month-old infants were able to successfully detect a reversal in the ordinal direction of a non-numerical, size-based sequence, but failed to detect the reversal in the ordinal direction of a numerical sequence in Experiments 1 and 2. In sum, the results of these experiments suggest that between 9 and 11 months of age, a change occurs to support ordinal numerical judgments.

Cantrell, L., & Smith, L. B. (2013). Open questions and a proposal: A critical review of the evidence on infant numerical abilities. ***Cognition, 128*****(3), 331–352. doi:10.1016/j.cognition.2013.04.008**

This article offers a critical review of the extant literature discussing infant numerical abilities. This review includes infants' numerical discriminations; detecting differences visually and auditorily; infants' small set tracking, visual working memory, object representation, and knowledge of more; infants' calculations and ordinal relations; number abstraction; and infants' intermodal matching.

This article also challenges the consensus view of a distinct and discrete quantity system by exploring consistencies and inconsistencies within the data surrounding controlling and isolating dimensions, small and large set discrimination, procedures and dependent measures of discrimination, and correlated dimensions. Finally, the researchers propose the Signal Clarity hypothesis as a guide for understanding infant quantity representation. This hypothesis suggests that the fundamental problem for infants is that they must discover that discrete quantity is the relevant task dimension and that stimulus properties that make number more perceptually salient are key to performance.

Ceulemans, A., Titeca, D., Loeys, T., Hoppenbrouwers, K., Rousseau, S., & Desoete, A. (2015).

The sense of small number discrimination: The predictive value in infancy and toddlerhood for numerical competencies in kindergarten. *Learning and Individual Differences, 39*, 150–157. doi:10.1016/j.lindif.2015.03.009

This study explores the following three research objectives: (a) whether performance on habituation tasks relate to numerical competencies in kindergarten (NCK), (b) whether infants' and toddlers' number discrimination performances predict NCK, and (c) whether number discrimination performance at 8 and 24 months of age correlated significantly and whether it could be considered a stable measure throughout development. To address these research objectives, a cohort of Belgian families agreed to have their child participate in the study at 8, 24, and 48 months of age. At 8 and 24 months, children's number discrimination was assessed (at 8 months via habituation and at 24 months via a manual search task). At 48 months, children's counting, arithmetic operations, and cardinality knowledge were assessed. The results indicated that no observable relationship was found between number discrimination in infancy and NCK, but that a relationship between toddler's small number discrimination and NCK exists. This suggests that toddler number discrimination performance may be able to predict at-risk mathematical development.

Clearfield, M. W., & Mix, K. S. (1999). Number versus contour length in infants' discrimination of small visual sets. ***Psychological Science*****,** ***10*****(5), 408–411.**

This article explores the role that continuous variables, such as area and contour length, play in infant numerical abilities. In this study, researchers habituated half of 16 infants between 6 and 8 months of age to two squares, and habituated the other half to three squares. Next, the infants were presented with trials that offered changes in number, as well as contour length. The results of this study indicated that the infants dishabituated to the changes in contour length when the number remained constant, but did not dishabituate to number when the contour length remained unchanged. These results suggest that infants attend to continuous quantity (e.g., contour length) rather than number and that perceptual variables may have been underestimated in previous studies addressing infants' numerical abilities.

Clements, D. H. (1989). Consensus, more or less. *Journal for Research in Mathematics Education, 20*(1), 111–119.

This article is a review of two books in the area of early counting and number concepts. The first book, *Construction of Arithmetical Meaning and Strategies* by L. P. Steffe and P. Cobb, highlights the development of first and second graders' counting structures. The second book reviewed is K. C. Fuson's *Children's Counting and Concepts of Number* (a resource listed in this annotated bibliography).

Clements, D. H., & Sarama, J. (2014). *Learning and teaching early math: The learning trajectories approach.* **(2nd ed.). New York City: Routledge.**

Sarama, J., & Clements, D. H. (2009). Early childhood mathematics education research: Learning trajectories for young children. New York City: Routledge.

These books review research on mathematics, thinking, learning, and development for children from birth to age 8. The relevant chapters for number include chapters 2–6 in both books (they are complementary chapters—the 2009 book emphasizes detailed descriptions of the research and the 2014 book summaries this and emphasizes teaching and curricular activities). Included within these books are charts with developmentally appropriate activities for children across each learning trajectory. Thus, readers can see the developmental level of thinking *and* associated activities for each age. Each chapter (especially in the 2014 book) provides practical tips and strategies for teachers. Highlights of these chapters are in the introduction to this section.

Clements, D. H., Sarama, J., & DiBiase, A.-M. (2004). Engaging young children in mathematics: Standards for early childhood mathematics education. Mahwah, NJ: Erlbaum.

This book, a report of a conference on early math standards funded by the National Science Foundation and the Exxon-Mobil Foundation, constitutes a set of guidelines that enable standards writing groups to create consistent and complementary (and, as much as possible, common) standards that are based on a current understanding of research, practice, and policy in early childhood mathematics education. The first section, "Major Themes and Recommendations," summarizes the research findings and guidelines. Some of the conclusions relevant to the youngest children include the following (we include number especially because research shows that it has a large spatial component in very young children's cognition):

- Infants "habituate"—are sensitive to—differences between sets of one to three items.
- Toddlers can identify whether collections are the "same" number or which is "more" visually.
- By 24 months of age, many toddlers have learned their first number word (typically "two").

- Depending on the early environment, children begin to try to count using verbal number names at 2 or 3 years of age.
- Toddlers understand “first” and “last.”
- The easiest type of collection for 3-year-olds to count has only a few objects that are arranged in a straight line and can be touched as children proceed with their counting.
- Toddlers can trade several small items for a larger one, which is an initial step to understanding grouping and, eventually, place value.
- Most preschool students can “see” that two items and one item make three items. Even 3-year-olds can solve problems such as “one and one more” nonverbally (i.e., by creating a matching collection).
- Partitioning is the operation of decomposing a set of objects into sets of equal sizes. The simplest form of this big idea is readily understandable by children. It emerges at around 3 years of age.

Feigenson, L., Carey, S., & Hauser, M. (2002). The representations underlying infants’ choice of more: Object files versus analog magnitudes. *Psychological Science*, *13***(2), 150–156. doi:10.1111/ 1467-9280.00427**

This article explores infants’ representation of quantity, specifically more and less. Through the use of three experiments, the researchers assessed 10- and 12-month-old infants on their ability to select more. The infants watched an examiner place crackers into opaque containers and were then allowed to crawl toward their container of choice to retrieve the crackers. The results indicated that the infants were able to select the container with more crackers in comparisons of one versus two crackers and two versus three, but that they were unable to select the container with more crackers in comparisons of three versus four, two versus four, and three versus six. These results suggest that the infants utilized object-file representation to make their selections.

Feigenson, L., Carey, S., & Spelke, E. (2002). Infants’ discrimination of number vs. continuous extent. *Cognitive Psychology, 44***(1), 33–66. doi:10.1006/cogp.2001.0760**

This article discusses seven experiments that explore infants’ representation of the cardinal values of small sets of objects while controlling for continuous stimulus properties, including surface area, volume, and contour length. Several of these experiments were modeled after or were extensions of previous studies that target infant sensitivity to continuous stimulus dimensions when encoding a number of objects in an array. The results indicate that infants, when presented with a small set of objects, did not demonstrate sensitivity to number in the habituation or transformation paradigms when continuous extent was controlled for.

Fuson, K. C. (1988). ***Children's counting and concepts of number*. New York City: Springer-Verlag.**

Fuson's goal was to describe the results of research in the area of children's counting and numerical thinking, and the ways that this thinking changes over 2–8 years of age. Her second goal was to provide a framework within which future research could function. Fuson suggests a comprehensive categorization structure for correspondence errors (i.e. errors in establishing the one-to-one correspondence between number words and objects) and children's knowledge about relationships between counting and cardinality (including the early formation of such relationships). Fuson concludes that children follow different routes toward building a relationship between counting and cardinality.

Gallistel, C. R., & Gelman, R. (1992). Preverbal and verbal counting and computation. ***Cognition, 44*(1–2), 43–74. doi:10.1016/0010-0277(92)90050-R**

This article is included in this annotated bibliography because it provides a detailed overview of how a preverbal system of counting and arithmetic reasoning is linked between human and animal cognition. A thorough overview of numerical competence in animals and the development of adult numerical competence is provided and offers a context within which to place infant and toddler mathematical development.

Geist, K., & Geist, E. (2008). Do re mi, 1-2-3: That's how easy math can be—Using music to support emergent mathematics. ***Young Children*, 63(2), 20–25. Retrieved from http://0-search.proquest.com.bianca.penlib.du.edu/docview/61969123?accountid=146086**[6]

This article discusses the relationship between math and music in the brain for infants and toddlers. Practical suggestions are provided for teachers and caregivers to incorporate with regard to rhythm, steady beats, melody, tempo, dynamics, timbre, and style. This article emphasizes that teachers do not need to be musicians in order to incorporate musical techniques in the classroom, and that music can be an ideal tool for supporting early mathematical thinking. Specific to number sense, this article touches on the relationship between music and one-to-one correspondence.

Gelman, R., & Gallistel, C. R. (1978). ***The child's understanding of number.* Cambridge, MA: Harvard University Press.**

In this influential work, the authors report the results of some half dozen years of research into when and how children acquire numerical skills. They provide a new set of answers to these questions, and overturn much of the traditional wisdom on the subject. This book brings forth the notion that children adhere to the sequence aspects of counting (i.e., the "stable-order principle"), later to the correspondence aspects (the "one-to-one principle"), and finally to the cardinality

[6] Please note that free access is not available to this document through the cited website.

aspects of number. Chapters include “Focus on the Preschooler,” “Training Studies Reconsidered,” “More Capacity Than Meets the Eye: Direct Evidence,” “Number Concepts in the Preschooler?,” “What Numerosities Can the Young Child Represent?,” “How Do Young Children Obtain Their Representations of Numerosity?,” “The Counting Model,” “The Development of the How-To-Count Principles,” “Reasoning About Number,” “Formal Arithmetic and the Young Child’s Understanding of Number,” and “What Develops and How.”

Greenberg, J. (2012). More, all gone, empty, full: Math talk every day in every way. *Young Children, 67***(3), 62–64. Retrieved from**
https://www.naeyc.org/yc/files/yc/file/201205/RockingAndRolling_YC0512.pdf

This brief and practical article provides suggestions for how teachers and caregivers can incorporate math talk with infants and toddlers in everyday living. The author gives examples of how to pull in the math concepts of number and operation; shapes and spatial relationships; measurement; patterns, relationships, and change; and collecting and organizing information into everyday activities and conversations. Specific to number, this article outlines the importance of understanding the concept of number, quantity, order, ways of representing numbers, one-to-one correspondence, and counting, and provides examples of how to integrate these concepts into the classroom.

Jordan, K. E., Suanda, S. H., & Brannon, E. M. (2008). Intersensory redundancy accelerates preverbal numerical competence. *Cognition***, 108(1), 210–221.**
doi:10.1016/j.cognition.2007.
12.001

This article explores the question of whether 6-month-old infants are able to make more accurate numerical discriminations when they are provided with the same information about number through multiple sensory modalities. To test their research question, the researchers conducted two experiments. In Experiment 1, infants were habituated to a video of a ball bouncing either 8 or 12 times, and a tone was emitted every time the ball made contact with the surface. The infants were then shown alternating videos of the ball bouncing either 8 or 12 times. Experiment 2a was conducted with a similar methodology as Experiment 1; however, the auditory tone was omitted. Experiment 2b replaced the auditory tone with classical music, which provided multisensory, non-numeric information. This research suggests that infants are only able to discriminate numerosities with a 2:3 ratio during multimodal audiovisual events.

Li, X., & Baroody, A. J. (2014). Young children’s spontaneous attention to exact quantity and verbal quantification skills. *European Journal of Developmental Psychology, 11***(5), 608–623.**
doi:10.1080/17405629.2014.896788

This article begins with a presentation of the theoretical framework that the researchers utilized in developing their study. Specifically, the researchers discuss whether young children can attend to the exact quantity of small collections and whether young children can attend to exact quantity without the corresponding verbal number knowledge. The research study involved having children 2.5–3.5 years of age complete one non-verbal matching task and two verbal qualification tasks. The non-verbal matching required the children to put the same number of items on a paper mat as the examiner had on a paper mat using homogenous (e.g., blue T-Rexes), semi-homogenous (e.g., one blue T-Rex and one yellow T-Rex), and heterogeneous (e.g., one blue T-Rex, one red Stegosaurus, and one yellow Triceratops) trials. The verbal qualification tasks included "point to *n*" and "how many?" tasks. The results indicated that children 2.5–3.0 years of age performed better with two than with three than the children 3.0–3.5 years of age with both heterogeneous and homogeneous collections.

Linnell, M. & Fluck, M. (2001). The effect of maternal support for counting and cardinal understanding in pre-school children. ***Social Development, 10*****, 202–220. doi:10.1111/1467-9507.00159**

This longitudinal study examines the role that mother–child interaction plays in the development of counting and cardinality in young children. Data were collected by observing 18 pairs of mothers and their children performing a variety of tasks (e.g., unassisted or assisted counting, "give *x*" task) when the child was 32, 38, and 44 months of age. The results indicated that the way the mothers focused their child's attention on objects in a set differed between tasks even though maternal support for the word count sequence was similar. Additionally, at 32 and 38 months of age, children were more successful in counting tasks rather than "give *x*" tasks, which is consistent with other research that suggests counting and cardinality are not inherently linked.[7]

Lipton, J. S., & Spelke, E. S. (2003). Origins of number sense: Large-number discrimination in human infants. ***Psychological Science*****,** ***14*****(5), 396–401.**

This study investigates infants' sensitivity to numerosity in auditory sequences. The researchers conducted four experiments to better understand 6-month-old infants' numerosity discrimination over two different ratios (16 versus 8 sounds and 12 versus 8 sounds) and 9-month-old infants' discrimination of sounds over two different ratios (8 versus 12 sounds and 8 versus 10 sounds). The findings from this study suggested that 6-month-old infants are able to discriminate 16 from 8 sounds, but are unable to discriminate 12 from 8 sounds. The 9-month-old infants were able to discriminate 12 from 8 sounds, but were unable to discriminate 10 from 8 sounds. These results suggest that infant numerosity discrimination increases in precision over development.

---

[7] Fluck, M. (1995). Counting on the right number: Maternal support for the development of cardinatlity. *The Irish Journal of Psychology, 16, 2*, 133-149. Siegler (1991). In young children's counting procedures precede principles. *Educational Psychology Review, 3*, 127-135. Wynn (1990). Children's understanding of counting. *Cognition, 36*, 155-193. Wynn (1992). Children's acquisition of the counting words in the number system. Cognitive Psychology, 24, 220-251.

Mazzocco, M. M. M., Feigenson, L., & Halberda, J. (2011). Preschoolers' precision of the approximate number system predicts later school mathematics performance. *PloS ONE, 6***(9), e23749.**

This article discusses the Approximate Number System (ANS), which is defined as a mental system of magnitude representations that produces an intuitive "number sense" across species and throughout human development, starting from just after birth. The study described in this article assesses whether ANS precision prior to entering school predicts school mathematics performance during or after kindergarten. This study required preschool children to complete an assessment involving ANS numerical discrimination at three time points. This article is included in this annotated bibliography because the study it discusses is the first to show that ANS precision measured in years prior to formal schooling predicts mathematics ability in primary school.

Mix, K. S., Levine, S. C., & Huttenlocher, J. (2002). *Quantitative development in infancy and early childhood***. New York City: Oxford University Press.**

This book offers an overview of early quantitative development from infancy through the early school years. The chapters in this book include "Historical Trends and Current Issues," "Quantification in Infancy," "Quantification in Early Childhood," "Quantification of Discrete Sets: A Synthesis," "Continuous Amount," "Relative Quantity," "Nonverbal Representation of Quantity," "Acquiring Conventional Skills," and "The Whole Child."

Sarnecka, B. W., & Lee, M. D. (2009). Levels of number knowledge during early childhood. *Journal of Experimental Child Psychology, 103***(3), 325–337. doi:10.1016/j.jecp.2009.02.007**

This article analyzes children's (ages 2–4) incorrect responses on "give *n*" tasks. A "give *n*" task asks a child to produce a set number of items (e.g., "Give me two blocks") and errors are typically attributed either to guessing or performance errors. The results from this study indicate that most wrong answers on "give *n*" tasks are guesses rather than counting or estimation errors. This suggests that number development is discontinuous and involves conceptual change.

Starkey, P., & Cooper, R. G. (1980). Perception of numbers by human infants. *Science, 210***(4473), 1033–1035. doi:10.1126/science.7434014**

This study aimed to provide evidence that 22-week-old infants are able to discriminate exact numbers of items. Using a habituation-dishabituation paradigm, researchers discerned that infants are able to discriminate, represent, and remember small numbers of items, but not for a large number of items. The researchers hypothesize that these results are potentially due to the infants' subitizing and suggest that these data indicate that verbal counting has precursors during infancy.

Starkey, P., Spelke, E. S., & Gelman, R. (1990). Numerical abstraction by human infants. *Cognition, 36***(2), 97–127. doi:10.1016/0010-0277(90)90001-Z**

The experiments presented in this article were performed to investigate whether infants can detect numerical correspondences between different kinds of items (e.g., visible objects, auditory events). Across a variety of experiments, researchers found that human infants ages 6–8 months were able to detect numerical correspondences, such as relation, between item sets presented in different sensory modalities. These findings suggest that early numerical abilities can develop in infants before the development of language, complex actions, or cultural experiences with number.

Strauss, M. S., & Curtis, L. E. (1981). Infant perception of numerosity. ***Child Development*****, 52, 1146–1152.**

This research study explores whether infants 10–12 months of age are able to discriminate between visual arrays of different amounts (e.g., two versus three, three versus four, four versus five items). The experiment was set up with infants in one of two conditions. Infants in the heterogeneous condition were habituated to a series of slides where the number of items remained constant, but the type of items (e.g., dogs, houses) and their position on the card changed. In the homogenous condition, the item type (chicks) did not change, but the size and position of the items did change. The researchers found that the infants were able to discriminate between two and three items, but were unable to discriminate between four and five items. An interesting finding in this study was that for the three versus four discrimination, males were able to make the discrimination in the heterogeneous condition whereas females were able to make the discrimination in the homogenous condition. The researchers concluded that because the infants in this study were preverbal, these results indicate that early counting skills are preceded by an awareness of numerosity.

Wynn, K. (1992). Addition and subtraction by human infants. ***Nature*****, 358(6389), 749–750. doi:10.1038/358749a0**

This article outlines an experiment that required infants (4–5 months of age) to be divided into two separate groups. The first group (the 1+1 group) was shown an item being presented in a display case and then covered up. Next, a second item was added to the display cause. It, too, was covered up. Then, the screen hiding the items was removed so that the infants could see how many items were in the display case. Sometimes the number of items left in the display case was correct, and sometimes it was incorrect. The second group (the 2–1 group) followed the same methodology as group one; however, items were removed instead of added to the display case. The results showed that infants looked longer at incorrect rather than correct results, suggesting that infants understand certain numerical concepts and that mathematical abilities are an innate part of human development.

Wynn, K. (1992). Children's acquisition of the number words and the counting system. *Cognitive Psychology, 24***(2), 220–251. doi:10.1016/0010-0285(92)90008-P**

The purpose of this study was to examine the development of children's understanding of number words. This article discusses a longitudinal study of 2- and 3-year-olds that took place over a 7-

month period. The children were given four tasks: Give-a-Number, How-Many, Color Control, and Point-to-*x.* Their answers to each of these tasks were analyzed over time, and the results suggest that it takes children approximately 1 year after understanding that number words refer to numerosities to learn how the counting system represents numerosity.

Wynn, K. (1998). Numerical competence in infants. In C. Donlan (Ed.), *The development of mathematical skills***. Hove, England: Taylor & Francis.**

This chapter describes the preexisting numerical capacities that humans are innately equipped with, and discusses the types of mathematical abilities that human infants are capable of. The similarities between humans and animals in terms of numerical representation and review are presented, and a discussion surrounding a mental mechanism specific to number in both human infants and animals is provided.

Wynn, K. (1998). Psychological foundations of number: Numerical competence in human infants. *Trends in Cognitive Sciences, 2***(8), 296–303. doi:10.1016/S1364-6613(98)01203-0**

This article provides an overview of research describing the numerical abilities of human infants. Evidence is provided to support that infants can engage in numerical computation, two alternative accounts of infants' numerical competence are provided, and evidence of the difficulty with the concept of zero is presented. This article concludes with support for the hypothesis that humans possess a specialized mental mechanism for number that has evolved over time through natural selection.

Xu, F. (2003). Numerosity discrimination in infants: Evidence for two systems of representations. *Cognition, 89***(1), B15–B25. doi:10.1016/S0010-0277(03)00050-7**

This article discusses two experiments designed to investigate the number discrimination abilities of 6-month-old infants using two sets of numbers (four versus eight and two versus four), with two continuous variables controlled for (total filled area and total contour length). In Experiment 1, infants were habituated to either four discs or eight discs, and were then presented with four and eight discs alternately. The discs in the four-disc array had twice as large a contour length as the discs in the eight-disc array, and the total filled area was controlled for. The results indicated that the infants were able to successfully discriminate the four-disc array from the eight-disc array. In Experiment 2, the researchers used the same methodology but asked the infants to discriminate between two and four discs. The infants in Experiment 2 were unable to discriminate between the two-disc and four-disc arrays. These results suggest that infants can discriminate between large numbers (when continuous variables are controlled) and are unable to discriminate between small numbers (when continuous variables are controlled).

Xu, F., & Spelke, E. S. (2000). Large number discrimination in 6-month-old infants. *Cognition, 74***(1), B1–B11. doi:10.1016/S0010-0277(99)00066-9**

This article outlines two studies that explore how 6-month-old infants represent approximate numerosities. In Experiment 1, the researchers had the infants habituate to either an array of 8 dots or an array of 16 dots that varied in size and position, and then alternated showing them 8- or 16-dot arrays. The results of Experiment 1 suggest that infants are able to differentiate between 8- and 16-element displays (when continuous variables are controlled). In Experiment 2, the methodology was the same as in Experiment 1; however, the ratio changed to 1:3 and the infants had to discriminate between 8 and 12 dots. The results of Experiment 2 evidenced that the infants were unable to differentiate between large numerosities with a reduced ratio. The results of these two studies suggest that the infants' responses were based on true representations of number and that infants can discriminate between large numerosities only when the ratio difference between the two numerosities is large.

Xu, F., Spelke, E. S., & Goddard, S. (2005). Number sense in human infants. ***Developmental Science, 8*****(1), 88–101. doi:10.1111/j.1467-7687.2005.00395.x**

This article presents four experiments that examine 6-month-old infants' capacity to represent numerosity in visual-spatial displays. These experiments aim to explore the ratio limit in infants with higher numerosities (i.e., 16 versus 32 and 16 versus 24) and the discrepancy between large-number and small-number numerosities. Experiment 1 involved infants being habituated to either 16 or 32 discs, and then shown displays of 16 or 32 discs intermittently. The results indicated that the infants were able to discriminate between visual arrays of much larger numbers than typically investigated. Experiment 2 replicated the methods of Experiment 1, but required the infants to discriminate between arrays of 16 and 24 discs. The infants had much more difficulty discriminating between arrays of 16 and 24 discs, which indicates that infants discriminate between sets that differ by a ratio of 2.0 but not 1.5. Experiment 3 also followed the same method as Experiment 1; however, the sizes of the discs were twice as large in the 16-disc array as in the 32-disc array. The results of Experiment 3 suggest that the infants were able to discriminate differences in numerosities despite changes in contour length. Finally, Experiment 4 tested the infants' ability to discriminate between one and two discs. The results from Experiment 4 suggest that infants were less able to discriminate between one and two discs than between 16 and 32 discs. Collectively, these results indicate that infants are able to discriminate between large numerosities even when changes in continuous variables are considered.

Problem Solving and Prediction

Introduction

When children learn to pull on a blanket to bring a toy into their reach, they are engaging in means-end problem solving, which may begin to emerge between 6 and 9 months of age. In general, considering their limited experience, young children are impressive problem solvers. Young children have multiple problem-solving strategies at their disposal, as well as the ability to choose among them. Means-end analysis is a general strategy as are several others. Children know and prefer cognitively easier strategies. Trial and error, with light cognitive requirements, begins early, with Piagetian circular reactions trying to make an interesting sight or sound repeat. These strategies develop throughout the toddler and preschool years, enabling children to address problems of increasing complexity. By the time that children are in kindergarten, they can solve a wide range of addition, subtraction, multiplication, and division problems when they are encouraged to use manipulatives or drawings to model the objects, actions, and relationships in those situations. Reasoning from domain-specific knowledge builds upon the basis of mindful general problem-solving and reasoning abilities that are evident from the earliest years.

References on Problem Solving and Prediction

Clements, D. H., & Sarama, J. (2014). *Learning and teaching early math: The learning trajectories approach.* **(2nd ed.). New York City: Routledge.**

Sarama, J., & Clements, D. H. (2009). Early childhood mathematics education research: Learning trajectories for young children. New York City: Routledge.

These books review research on mathematics, thinking, learning, and development for children from birth to age 8. There is much material on problem solving across nearly all chapters in the books; however, chapter 13 in both books emphasizes problem solving and reasoning specifically. Each chapter (especially in the 2014 book) provides practical tips and strategies for teachers. Highlights of these chapters are in the introduction to this section.

Denison, S. M. (2012). Inductive Inference in Infants and Young Children: The Role of Probabilistic Reasoning. University of California, Berkeley: Psychology. Retrieved from http://escholarship.org/uc/item/4ff9g022

Denison's dissertation explores the ability of infants to make probabilistic inferences in various experiments. The first set of experiments suggests that infants must be at least 6 months old in order to make generalizations about the likely composition of a large population after observing the contents of a small sample drawn from that population. The second set of experiments suggests that infants can sometimes demonstrate greater competence than adults in making probabilistic inferences. The third set of experiments suggests that infants can use single-event probability computations to make predictions about where to direct their search in order to locate a desired object. This dissertation concludes with the suggestion that infants and young children do make probabilistic inferences, although this skill is early emerging and not consciously reflected. Moreover, the author suggests that young learners are capable of making rapid inductive inferences by capitalizing on their ability to compute probabilities in order to acquire knowledge in a variety of domains, including causal reasoning.

Denison, S., & Xu, F. (2010). Twelve- to 14-month-old infants can predict single-event probability with large set sizes. *Developmental Science, 13***(5), 798–803. doi:10.1111/j.1467-7687.2009. 00943.x**

This study questions whether infants can make predictions regarding single-event probability with large set sizes. Each infant completed two trials. The first trial was a preference trial to determine whether they preferred pink or black lollipops. The second trial was the test trial in which infants where shown two jars: one filled mostly with pink lollipops and one filled mostly with black lollipops. Next, the experimenter selected one lollipop from each jar and placed them in two separate cups. The results indicated that 78 percent of infants began to reach for and search the cup that had the lollipop from the jar with their preferred color of lollipops. These results suggest that infants have the ability to reason about single-event probabilities.

Richards, D. D., & Siegler, R. S. (1981). Very young children's acquisition of systematic problem-solving strategies. *Child Development, 52***(4), 1318–132.**

The aim of this study was to identify the transition in early childhood from less to more systematic problem-solving strategies. This study examined two experiences that may be involved in the developmental progression of problem-solving strategies: (a) encouragement from other people to be more analytical, and (b) encountering a problem on an identical/similar task that had large differences within a dimension. Forty 3-year-olds were shown problems involving pegs, weights, and a balance scale, and were asked to make predictions on which side would go down if a lever was released. The results indicated that encouragement to take an analytical attitude helped children select a systematic strategy to solve the balance scale problem, and to use that same strategy again 1 week later. Additionally, it was found that almost all children who utilized a systematic strategy adopted a rule 1', in which they would arbitrarily choose a side when two sides had equal amounts of weight, rather than rule 1, in which they would predict that if one side of the scale had more weight it would go down, and otherwise the scale would balance.

Téglás, E., Girotto, V., Gonzalez, M., & Bonatti, L. L. (2007). Intuitions of probabilities shape expectations about the future at 12 months and beyond. *Proceedings of the National Academy of Sciences of the United States of America, 104***(48), 19156–19159. doi:10.1073/pnas.0700271104**

This research aimed to understand whether infants have expectations about future, unseen events based on their likelihood. The study involved showing infants movies in which three identical objects and a dissimilar object bounced inside a container, a person hiding the container, and a pipe ejecting one of the four objects. The results demonstrated that infants looked significantly longer at improbable outcomes (the dissimilar object being ejected from the pipe rather than one of the three similar objects being ejected from the pipe) than at probable outcomes. The researchers manipulated the experiment to prohibit certain objects from leaving, and the results were similar. The implications for research are presented.

Xu, F., & Denison, S. (2009). Statistical inference and sensitivity to sampling in 11-month-old infants. *Cognition, 112***(1), 97–104. doi:10.1016/j.cognition.2009.04.006**

This research addresses the question of whether infants can integrate intentional information in statistical inference tasks. The study designed to address this question involved 11-month-old infants in one of three conditions (random sampling, non-random sampling, blindfolded) and an experimenter pulling red or white balls out of a large box with either mostly white or mostly red balls. The results indicated that the infants were sensitive to whether a sample was randomly drawn from a population or not, and that they take into account intentional information when determining the relationship between samples and populations. These results suggest that domain-specific knowledge is integrated with statistical inference mechanisms early in development.

Caregivers (Parents and Providers)

Introduction

Of course, families play a major role in young children's development, including their learning of mathematics.[8] There is a relationship between the frequency with which parents use number and their children's early mathematical performance.[9] However, there are several sociocultural barriers. For example, although parents believe that both home and school are important for reading development, they consider the school more important for mathematics development. As a result, they provide fewer math experiences at home compared to reading.[10] They believe that it is more important to help their children learn literacy than mathematics.[11] They prefer teaching language and they believe that language is more important to learn than mathematics.[12] More for the learning of language than mathematics, parents believe that pedagogy should consist of determination to ensure that children acquire specific knowledge, delving deeply into children's understanding and facilitating children's learning in their everyday lives. These are profound differences that have severe implications.

Furthermore, both parents and providers have a limited view of the breadth of mathematics appropriate for young children.[13] They know more about what might be taught in language than

8 (adapted from Clements, D. H., & Sarama, J. (2014). *Learning and teaching early math: The learning trajectories approach* (2nd ed.). New York, NY: Routledge. pp. 264-265)

9 Blevins-Knabe, B., & Musun-Miller, L. (1996). Number use at home by children and their parents and its relationship to early mathematical performance. *Early Development and Parenting, 5*, 35-45. doi: 10.1002/(SICI)1099-0917(199603)5:1<35::AID-EDP113>3.0.CO;2-0

10 Sonnenschein, S., Baker, L., Moyer, A., & LeFevre, S. (2005, April). *Parental beliefs about children's reading and math development and relations with subsequent achievement.* Paper presented at the Biennial Meeting of the Society for Research in Child Development, Atlanta, GA.

11 Cannon, J., Fernandez, C., & Ginsburg, H. P. (2005, April). *Parents' preference for supporting preschoolers' language over mathematics learning: A difference that runs deep.* Paper presented at the Biennial Meeting of the Society for Research in Child Development, Atlanta, GA.

12 Id.

13 Sarama, J. (2002). Listening to teachers: Planning for professional development. *Teaching Children Mathematics, 9*, 36-39.

mathematics.[14] This is true regardless of whether the parents are Hispanic or not, and whether they are low or middle socio-economic status. However, cultural differences are occasionally relevant. For example, mothers in China are more likely than mothers in the United States to teach arithmetic calculation in their everyday involvement with children's learning, and maternal instruction was related to Chinese, but not U.S., children's learning of proportional reasoning.[15] Mothers in China rate mathematics as equal in importance to reading, but mothers in the United States rate mathematics as less important than reading.[16]

The more mathematics activities that parents and providers engage their children in, the higher the children's achievement.[17] In all contexts, early childhood providers should be strong advocates for foundational and explicit mathematical experiences for all children, of all ages. In the earliest ages especially, these often can be seamlessly integrated with children's ongoing play and activities; however, this usually requires a knowledgeable adult who creates a supportive environment and provides challenges, suggestions, tasks, and language. Programs designed to improve home mathematics learning have been found to be most successful when they had three components: joint and separate sessions for parents and children, a structured numeracy curriculum, and "bridging" activities for parents to develop their child's numeracy at home.[18]

References on Caregivers (Parents and Providers)

Clements, D. H., & Sarama, J. (2014). ***Learning and teaching early math: The learning trajectories approach*****. (2nd ed.). New York City: Routledge.**

Sarama, J., & Clements, D. H. (2009). Early childhood mathematics education research: Learning trajectories for young children. New York City: Routledge.

These books review research on mathematics, thinking, learning, and development for children from birth to age 8. There is much material on what parents and providers can do across nearly all chapters in the books; however, chapters 15 and 16 in the 2014 book discuss early children

---

[14] Cannon, J., Fernandez, C., & Ginsburg, H. P. (2005, April). *Parents' preference for supporting preschoolers' language over mathematics learning: A difference that runs deep.* Paper presented at the Biennial Meeting of the Society for Research in Child Development, Atlanta, GA.

[15] Pan, Y., Gauvain, M., Liu, Z., & Cheng, L. (2006). American and Chinese parental involvement inyoung children's mathematics learning. *Cognitive Development, 21*, 17-35.

[16] Miller, K. F., Kelly, M., & Zhou, X. (2005). Learning mathematics in China and the United States: Cross-cultural insights into the nature and course of preschool mathematical development. In J. I. D. Campbell (Ed.), *Handbook of mathematical cognition* (pp. 163-178). New York, NY: Psychology Press.

[17] Blevins-Knabe, B., & Musun-Miller, L. (1996). Number use at home by children and their parents and its relationship to early mathematical performance. *Early Development and Parenting, 5*, 35-45. doi: 10.1002/(SICI)1099-0917(199603)5:1<35::AID-EDP113>3.0.CO;2-0
Blevins-Knabe, B., Whiteside-Mansell, L., & Selig, J. (2007). Parenting and mathematical development. *Academic Exchange Quarterly, 11*, 76-80.

[18] Doig, B., McCrae, B., & Rowe, K. (2003). A good start to numeracy: Effective numeracy strategies from research and practice in early childhood. Canberra ACT, Australia: Australian Council for Educational Research.

mathematics education contexts and curricula (chapter 15, see pp. 270–275 for a section devoted to parents and families) and instructional practices and pedagogical issues (chapter 16). In the 2009 book, chapter 14 is devoted to professional development for providers. Each chapter (especially in the 2014 book) provides practical tips and strategies for teachers. Highlights of these chapters are in the introduction to this section.

Cook, G. A., Roggman, L. A., & Boyce, L. K. (2012). Fathers' and mothers' cognitive stimulation in early play with toddlers: Predictors of 5th grade reading and math. *Family Science, 2*, **131–145. doi:10.1080/19424620.2011.640559**

The goal of this study was to examine the impact that mothers' and fathers' interactions with toddlers have on children's math and reading outcomes in the 5th grade. The research questions addressed in this study include the following: (a) Are mother–toddler and father–toddler interactions, in families with and without resident biological fathers, associated with children's math and reading scores in 5th grade? (b) Does father–toddler cognitive stimulation during play make contributions to children's math and reading scores in 5th grade, independent of mothers' cognitively stimulating interactions, child gender, and Early Head Start (EHS) program enrollment? and (c) Are the contributions of parents' early cognitive stimulation to children's later school outcomes mediated by children's early language and cognitive skills, independent of child gender and EHS program enrollment? Data were collected via observations of mother–toddler and father–toddler interactions at age 2 and from standardized child assessments at age 3 and in 5th grade. The results indicated that children's earlier cognitive ability was not a predictor of their later math ability for children who did not live with their biological fathers. For children who did not live with their biological fathers, it was only the mothers' parenting interactions that made contributions to their later math achievement.

Gunderson, E. A., & Levine, S. C. (2011). Some types of parent number talk count more than others: Relation between parents' input and children's number knowledge. *Developmental Science, 14*(**5), 1021–1032. doi:10.1111/j.1467-7687.2011.01050.x**

In this article, the researchers theorize that parent number talk about small sets of numbers should help children learn how to correctly label sets of one, two, and three, but that parent number talk may not be sufficient for children to learn the cardinal principle (i.e. the word "two" refers to two objects). The study presented in this article investigates which types of number talk are most predictive of children's later cardinal number knowledge by videotaping, transcribing, and analyzing home visits over five sessions when the child was 14, 18, 22, 26, and 30 months of age. The results indicate that parents vary in the amount and types of number talk they engage in. The amount of number talk devoted to counting or labeling sets of objects, especially sets of objects ranging from 4 to 10 items, was predictive of children's later cardinal number knowledge.

Levine, S. C., Suriyakham, L. W., Rowe, M. L., Huttenlocher, J., & Gunderson, E. A. (2010). What counts in the development of young children's number knowledge? *Developmental Psychology, 46***(5), 1309–1319. doi:10.1037/a0019671**

This article examines the exposure to number talk that children experience within a culture in order to determine whether variation in the amount of number talk is related to the development of cardinal number knowledge. This study involved parent–child dyads being visited by researchers and videotaped for 90 minutes during typical everyday activities every 4 months while the child was between 14 and 30 months of age. The videotapes were transcribed, coded, and analyzed for emergent themes. The results suggest that (a) number talk typically involved low numbers; (b) when the children were 30 months of age, most number talk involved counting and labeling cardinal value sets; (c) the most common type of parent number talk was labeling set size, followed by counting, and the most common type of child number talk was counting, followed by labeling set size; and (d) parent number talk is related to children's greater understanding of the cardinal values of number words.

Linnell, M. & Fluck, M. (2001). The effect of maternal support for counting and cardinal understanding in pre-school children. *Social Development, 10***, 202–220. doi:10.1111/1467-9507.00159**

This longitudinal study examines the role that mother–child interaction plays in the development of counting and cardinality in young children. Data were collected by observing 18 pairs of mothers and their children performing a variety of tasks (e.g., unassisted or assisted counting, "give *x*" tasks) when the child was 32, 38, and 44 months of age. The results indicated that the way that the mothers focused their child's attention on objects in a set differed between tasks even though maternal support for the word count sequence was similar. Additionally, at 32 and 38 months of age, children were more successful in counting tasks rather than "give *x*" tasks.

McKelvey, L. M., Bokony, P. A., Swindle, T. M., Conners-Burrow, N. A., Schiffman, R. F., & Fitzgerald, H. E. (2011). Father teaching interactions with toddlers at risk: Associations with later child academic outcomes. *Family Science, 2***, 146–155. doi:10.1080/19424620.2011.637710**

This longitudinal study examines the impact that fathers' teaching interactions in early childhood have on academic skills in middle childhood in low-income families. The research questions addressed in this study include the following: (a) What are the associations between fathers' teaching interactions at ages 2 and 3 and their children's academic outcomes at ages 5, 7, and 10? and (b) How does the quality of fathers' teaching interactions predict later cognitive outcomes, even after accounting for mothers' behaviors. Data were collected from 109 families and involved observing the fathers as they taught their 2- to 3-year-old child an age-appropriate and novel task. The results indicate that teaching interactions between fathers and their children at age 2 were related to child outcomes in language, literacy, and math.

Integrating Math Across the Day

Introduction

Young children learn through play and everyday experiences. The resources in this section illustrate strategies to highlight how caregivers can support children in learning math concepts through daily routines and experiences. For example, a young child might learn about the physical properties of objects, and about his own ability to manipulate them, by playing with a ball. He is also learning about the spatial relationships that exist between objects. The caregiver who recognizes this exploration and thinks about ways to support him can further his understanding of physical objects and space.

There are several types of play, such as sensorimotor/manipulative and symbolic/pretend (Monighan-Nourot, Scales, Van Hoorn, & Almy, 1987; Piaget, 1962). Sensorimotor play might involve rhythmic patterns, correspondences, and exploring materials such as blocks. Symbolic play can be further classified as constructive, dramatic, or rule governed. Materials such as sand, play dough, and blocks offer many rich opportunities for mathematical thinking and reasoning (Perry & Dockett, 2002). Caregivers might provide suggestive materials (cookie cutters), engage in parallel play with children, and raise comments or questions regarding shapes and the amount of things (e.g., making multiple copies of the same shape in play dough with the cookie cutters or transforming sand or play dough objects into one another). Play in perceptually oriented toddlers is enhanced by providing realistic objects. All children should also play with structured, open-ended materials. The use of blocks and interlocking plastic bricks is strongly linked to mathematical activity in general, and with pattern and shape in particular. In symbolic play, teachers need to structure settings, observe play for its potential, provide materials based on their observations (e.g., if children are comparing sizes, teachers might introduce objects with which to measure), highlight and discuss mathematics as it emerges within play, and ask questions such as “How do you know?” and “Are you sure?” with regard to the child’s answer or solution (van Oers, 1996). In these everyday experiences, it is critical that caregivers use mathematical language and help children discuss and think about the mathematics they learn.

References on Integrating Math Across the Day

Clements, D. H., & Sarama, J. (2014). *Learning and teaching early math: The learning trajectories approach.* **(2nd ed.). New York City: Routledge.**

Sarama, J., & Clements, D. H. (2009). Early childhood mathematics education research: Learning trajectories for young children. New York City: Routledge.

These books review research on mathematics, thinking, learning, and development for children from birth to age 8. There is much material on integrating math across the curriculum in nearly all chapters in the books; however, chapter 16 in both books emphasize integrating math across the curriculum. Each chapter (especially in the 2014 book) provides practical tips and strategies for teachers. Highlights of these chapters are in the introduction to this section.

Geist, K., & Geist, E. (2008). Do re mi, 1-2-3: That's how easy math can be—Using music to support emergent mathematics. *Young Children*, **63(2), 20–25. Retrieved from http://0-search.proquest.com.bianca.penlib.du.edu/docview/61969123?accountid=14608**[19]

This article discusses the relationship between math and music in the brain for infants and toddlers. Practical suggestions are provided for teachers and caregivers to incorporate rhythm, steady beats, melody, tempo, dynamics, timbre, and style. This article emphasizes that teachers do not need to be musicians in order to incorporate musical techniques in the classroom, and that music can be an ideal tool for supporting early mathematical thinking.

Geist, K., Geist, E. A., & Kuznik, K. (2012). The patterns of music: Young children learning mathematics through beat, rhythm, and melody. *Young Children, 67***(1), 74–79. Retrieved from https://www.naeyc.org/files/yc/file/201201/Geist_Patterns_of_Music_Jan012.pdf**

Research indicates that music is children's first patterning experience, and it can help them engage in mathematics even when they do not recognize the activities as being related to math. This article provides practical tips for how caregivers can use music to engage children in mathematics. Examples include maintaining a steady beat throughout the mathematics lesson; changing the beat's tempo and dynamics regularly; observing, listening, and responding to children's musical behaviors; and trying to keep the music and math activities concept based and open ended.

Honig, A. (2007). Play: Ten power boosts for children's early learning. *Young Children, 62***(5), 72–78. Retrieved from https://mymission.lamission.edu/userdata/mermelrd/docs/NAE51.pdf**

This article discusses 10 ways in which children learn through play, including building dexterity; social skills; cognitive and language skills; number and time concepts; spatial understanding; reasoning of cause and effect; clarification of pretend versus real; sensory and aesthetic appreciation; extended attention span, persistence, and sense of mastery; and emotional release and relief from separation anxiety. Practical suggestions are provided for teachers to use within the classroom, and the theoretical framework from which these suggestions are wrought are provided.

Powell, S. R., & Nurnberger-Haag, J. (2015). Everybody counts, but usually just to 10! A systematic analysis of number representations in children's books. *Early Education and Development, 26***(3), 377–398. doi:10.1080/10409289.2015.994466**

---

[19] Please note that free access is not available to this document through the cited website.

This article provides a systematic analysis of how trade books portray numbers and other early numeracy concepts. The research questions guiding this study were (a) Which numbers do children have the opportunity to learn from trade books about numbers? and (b) Which number representations do children have the opportunity to learn in trade books, and in what ways are those representations used? To address these questions, the researchers analyzed 160 trade books related to numbers and coded them in the following categories: numbers, number representation, and pictorial representation. The results indicated that 68 percent of the books included numbers less than or equal to 10, and only 12 percent of the books presented the number 0 in comparison to 90 percent of the books that presented the number 1. Skip counting and backward sequencing were also underrepresented. Additionally, of the three different representations (numeral, number word, and quantity), fewer than half of the books reviewed provided all three. The implications for practice are addressed.

Reifel, S. (1984). Block construction: Children's developmental landmarks in representation of space. ***Young Children*****, 40(1), 61–67. Retrieved from http://0-search.proquest.com.bianca.penlib.du.edu/docview/63373256?accountid=14608**[20]

This article discusses the value of building blocks for young children and describes developmental changes in their symbolic representation of space. The implications for activities to promote spatial representation are drawn. Additionally, the author outlines the value of utilizing building blocks with young children.

Sumpter, L., & Hedefalk, M. (2015). Preschool children's collective mathematical reasoning during free outdoor play. ***The Journal of Mathematical Behavior, 39*****(0), 1–10. doi:10.1016/j.jmathb.2015. 03.006**

This article begins with an overview of mathematics and play, as well as mathematical reasoning. Next, the researchers explain their interpretation of a video analysis conducted on video recordings from 17 visits to a Swedish preschool (children ages 1–5) over a period of 2 months. The researchers provide three episodes to evidence their findings and highlight three mathematical aspects: measuring an object, developing a concept, and estimating and deciding height.

[20] Please note that free access is not available to this document through the cited website.

About the Authors

Douglas H. Clements, Ph.D., is the Kennedy Endowed Chair in Early Childhood Learning and professor at the University of Denver. Previously a kindergarten teacher for 5 years and a preschool teacher for 1 year, he has conducted research and published widely in the learning and teaching of early mathematics and computer applications in mathematics education. His most recent interests are in creating, using, and evaluating a research-based curriculum and in taking successful curricula to scale using technologies and learning trajectories. He has published more than 128 refereed research studies, 22 books, 86 chapters, and 300 additional publications. His latest books detail research-based learning trajectories in early mathematics education: *Early Childhood Mathematics Education Research: Learning Trajectories for Young Children* and a companion book, *Learning and Teaching Early Math: The Learning Trajectories Approach* (Routledge). Dr. Clements has directed more than 30 funded projects, including those funded by the National Science Foundation and the U.S. Department of Education's Institute of Education Sciences. Dr. Clements was a member of President Bush's National Math Advisory Panel, convened to advise the administration on the best use of scientifically based research to advance the teaching and learning of mathematics and co-author of the Panel's report. He was also a member of the National Research Council's Committee on Early Mathematics and co-author of their report. He is one of the authors of the National Council of Teachers of Mathematics' *Principles and Standards in School Mathematics* and *Curriculum Focal Points.* Additional information can be found at http://du.academia.edu/DouglasClements, http://www.researchgate.net/profile/Douglas_Clements, and http://portfolio.du.edu/dclemen9.

Carrie Germeroth, Ph.D., is assistant director of research at the Marsico Institute for Early Learning and Literacy at the University of Denver. Dr. Germeroth conducts research in cognitive and social-emotional development and early childhood classroom quality. She currently serves as a member of the Colorado Department of Education's School Readiness Assessment and served as project manager on the development of Colorado's Early Learning and Development Guidelines, as well as the Pre-Kindergarten Standards for the state of North Dakota. Dr. Germeroth routinely provides professional development and technical assistance on early math and social-emotional development to early childhood education audiences nationally. She was a co-developer of the instructional materials and professional development for the Scaffolding Early Learning and Dynamic Coaching models for early childhood educators. Dr. Germeroth has published research in several peer-reviewed early childhood journals, including the *Journal of Applied Developmental Psychology*, *Early Child Development and Care*, *American Journal of Play*, and the *Phi Delta Kappan*, and is the lead author on the publication *Self-Regulation for Academic Success*, with the ASCD (formerly known as the Association for Supervision and Curriculum Development). Dr. Germeroth is also a certified observer in the Toddler and Pre-K CLASS™ Observation systems and provides technical assistance to programs seeking to improve teacher–child interactions with the instruments.

Brittany Sovran is a Ph.D. candidate in the Child, Family, and School Psychology program at the University of Denver. Ms. Sovran currently works as a research assistant at the Kennedy Institute in the Morgridge College of Education at the University of Denver. She has supported several early childhood mathematics education grants. Her research experience on these projects has focused on early childhood assessment, data analysis, and interpretation. Ms. Sovran has presented at local and national conferences on the role of dramatic play in children's development and she is a certified Pre-K CLASS™ observer.